NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science Honours in Applied Statistics	
QUALIFICATION CODE: 08BSHS	**LEVEL:** 8
COURSE CODE: STP801S	**COURSE NAME:** STOCHASTIC PROCESSES
SESSION: JULY, 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Prof Rakesh Kumar
MODERATOR:	Prof Lawrence Kazembe

INSTRUCTIONS

1. Attempt any FIVE questions. Each question carries equal marks.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1. (Total marks: 20)

(a) Classify the stochastic processes according to parameter space and state space using suitable examples. (15 marks)

(b) What is gambler's ruin problem. (5 marks)

Question 2. (Total marks: 20)

Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that probability of a rainy day following a dry day is 1/2.

(i) Develop a two-state transition probability matrix of the Markov chain. (5 marks)

(ii) Given that May 1, 2023 is a dry day, find the probability that May 3, 2023 is a rainy day. (15 marks)

Question 3. (Total marks: 20)

(a) Define the period of a Markov chain. Differentiate between periodic and aperiodic Markov chains. (10 marks)

(b) What is the nature of state 1 of the Markov chain whose transition probability matrix is given below:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(10 marks)

Question 4. (Total marks: 20)

(a) Find the steady-state probabilities of the Markov chain whose one-step transition probability matrix is given below: (15 marks)

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

(b) Differentiate between super-martingale and sub-martingale. (5 marks)

Question 5. (Total marks:20)

Suppose that the customers arrive at a service facility in accordance with a Poisson process with mean rate of 3 per minute. Then find the probability that during an interval of 2 minutes:

(i) exactly 4 customers arrive (ii) greater than 4 customers arrive

(iii) less than 4 customers arrive

(e^{-6} =0.00248) (20 marks)

Question 6. (Total marks:20)

(a) Prove that if the arrivals occur in accordance with a Poisson process, then the inter-arrival times are exponentially distributed. (10 marks)

(b) Derive the Kolmogorov forward equations for a continuouse-time Markov chain. (10 marks)

---------------------------------------END OF QUESTION PAPER---